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A model Hamiltonian of the normal state of cuprate superconducting materials

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Abstract. We propose a simple model Hamiltonian to describe the normal state of cuprate superconducting materials. In this model the holes with single occupation constraint introduced by doping move in an antiferromagnetic background of copper spins. Using this model Hamiltonian, we can better explain the magnetic and transport properties of the normal state.

Since the discovery of cuprate superconducting materials [1] there has been considerable controversy over the choice of the appropriate microscopic Hamiltonian. It is generally agreed now, however, that Anderson's starting point [2], namely strong on-site Coulomb interactions among a partially filled band of the Cu 3d level, is correct. Following this clue, Zhang and Rice [3] proposed a single-band effective Hamiltonian: the t–J model. The key point of their work is that the hybridization strongly binds a hole on each square of O atoms to the central Cu^{2+} ions in a similar way as a hole in the single-band effective Hamiltonian; two holes feel a strong repulsion against residing on the same square. At zero doping, the t–J model directly reduces to the quantum antiferromagnetic Heisenberg model. Substantial progress has recently been achieved in understanding the Heisenberg limit, both theoretically and experimentally [4–6]. The effects of doping, however, are still highly controversial. The gauge theory of the t–J model [7–9] gives a better description of the transport properties of the normal state of cuprate superconducting materials, but it is difficult to explain its anomalous magnetic behaviour, which is shown by nuclear magnetic resonance (NMR) and other experiments.

Recently, Sokol and Pines [10] used pure scaling considerations to show that experimental data [11–14] on the NMR spin-lattice relaxation rate T_1 and spin-echo decay rate T_{2G} in cuprates imply a quantum critical (QC), $z = 1$ (where z is a dynamic exponential), behaviour over an unexpectedly broad doping range, and at low temperatures a quantum disorder (QD) $z = 1$ behaviour. The crossover to the $z = 2$ regime occurs only in fully doped materials through a metal–insulator transition.

The unusual physical properties of the normal state may derive from its strongly antiferromagnetic correlation behaviour. The doping will destroy the long-range antiferromagnetic correlation, but the system still retains the short-range antiferromagnetic correlation behaviour, which induces the system to shift from the renormalized classical (RC) regime to QC or QD regimes. From current experimental data, we believe that the properties of the normal state of cuprates are determined by two different regions: one is the central region of the first Brillouin zone of the copper spins, another is its corner region, i.e. near the regions $Q = (\pm\frac{\pi}{a}, \pm\frac{\pi}{a})$ which mainly reflect the antiferromagnetic behaviour of the system. Current theoretical works only consider one region or the other [7–9, 15]. In this paper we use a simple model Hamiltonian that captures this key feature of cuprates, in order to study the magnetic and transport properties of their normal state. We study the following model Hamiltonian:

$$H = -t\sum_{\langle ij\rangle}\tilde{c}^{+}_{i\sigma}\tilde{c}_{j\sigma} + V_0\sum_{i}\hat{S}_i\cdot\hat{s}_i + J\sum_{\langle ij\rangle}\hat{S}_i\cdot\hat{S}_j \tag{1}$$

where $\hat{s}_i = \frac{1}{2}\tilde{c}^{+}_{i\alpha}\hat{\sigma}_{\alpha\beta}\tilde{c}_{i\beta}$, $\tilde{c}_{i\alpha} = (1-n_{i-\alpha})c_{i\alpha}$, $c_{i\alpha}(c^{+}_{i\alpha})$ is the hole operator which derives from the doping, and $\hat{S}_i$ is the spin operator at site i which represents the copper spin. Here we adopt an effective square lattice for O which identifies the Cu lattice; the holes introduced by the doping reside on the effective lattice sites, which have a single occupation constraint because of the strong repulsive interaction among the holes. The first term in (1) is the hole hopping term. The second term describes the Kondo interaction between the copper and hole spins, which derives from the hybridization between the holes on the Cu sites and the holes on the O sites. The last term describes the antiferromagnetic interaction among the copper spins, which is valid for both the doped and undoped cases. This model Hamiltonian can be seen as an effective Hamiltonian deriving from the three-band Hubbard model where the hybridization between the hole on the Cu site and the hole on the O site induces an exchange (Kondo) interaction; here we think of the holes on the Cu sites as being localized, and the holes on the O sites hop in an effective lattice which identifies the Cu sites. This model Hamiltonian describes a very simple physical picture in which holes with a single occupation condition move in an antiferromagnetic background. In the Kondo regime, the copper and hole spins bind into a Kondo singlet, which is similar to the Zhang and Rice singlet; it then has an effective hopping on different sites. We believe that this model Hamiltonian describes the same physical properties as the t–J model at some energy scale, but it can explain the NMR data better than the t–J model. Note that this model is different from the Kondo lattice model because of the single occupation condition of the holes; it is different from the t–J model because the hopping holes are induced by the doping.

We adopt a common method to deal with the single occupation condition by introducing a slave boson: $\tilde{c}_{i\sigma} = p^{+}_i f_{i\sigma} = b_i f_{i\sigma}$, $p^{+}_i p_i + f^{+}_{i\sigma}f_{i\sigma} = 1$ (or $b^{+}_i b_i = f^{+}_{i\sigma}f_{i\sigma}$). Here $b_i(= p^{+}_i)$ is a hard-core boson operator which describes the charge degree of the hole, and $f_{i\sigma}$ is a fermion operator that describes its spin degree. In the hole representation, the Hamiltonian in (1) can be written as

$$\begin{aligned} H = -t\sum_{\langle ij\rangle}(\eta^{*}_{ij}f^{+}_{i\sigma}f_{j\sigma} + \chi_{ij}b^{+}_i b_j) + V_0\sum_i b^{+}_i b_i\hat{S}_i\cdot\hat{s}_i + J\sum_{\langle ij\rangle}\hat{S}_i\cdot\hat{S}_j + t\sum_{\langle ij\rangle}\eta^{*}_{ij}\chi_{ij} \\ + \sum_i\lambda_i(b^{+}_i b_i - f^{+}_{i\sigma}f_{i\sigma}). \end{aligned} \tag{2}$$

Here we introduce two Hubbard–Stratonovich fields η^{*}_{ij} and χ_{ij} to decouple the hard-core boson and fermion, and λ_i is a Lagrangian multiplier that ensures the single occupation condition. To treat the hard-core nature of the bosons effectively, we make the following transformation to transform the hard-core bosons into fermions with a vortex tube carrying one flux quantum attached to each [16, 17]:

$$b^{+}_i = h^{+}_i\exp\Big(-\mathrm{i}\sum_{j\neq i}\theta_{ij}n_j\Big) \qquad n_j = h^{+}_j h_j \tag{3}$$

where the operators h^{+}_i, h_i obey Fermi statistics and θ_{ij} is the angle between the direction from site i to site j and some fixed direction, the x axis for example. We believe that in the low-temperature low-energy long-wavelength limits the hard-core nature of the bosons is important, so we must consider its effect.

In the spin and fermion coherent-state representations, for the spin part we take the antiferromagnetic Néel order as its background because, although the doping destroys long-range antiferromagnetic order, short-range antiferromagnetic order remains; for the hole part, we take the following approximations, i.e. we only consider the effect of the phase fluctuation:

$$\eta_{ij} = \langle\eta\rangle e^{iA_{ij}} \qquad \chi_{ij} = \langle\chi\rangle e^{iA_{ij}} \qquad \lambda_i = \lambda + iA_0(i)$$
$$A_{ij} = (\boldsymbol{r}_i - \boldsymbol{r}_j) \cdot \boldsymbol{A}\left(\frac{\boldsymbol{r}_i - \boldsymbol{r}_j}{2}\right). \tag{4}$$

It is well known that the Hamiltonian in (2) transforms into the following action:

$$\begin{aligned} S[a_\mu, \psi_h] = {} & \frac{1}{2g}\int_0^\beta d\tau \int d^2x \left((\nabla\hat{\Omega})^2 + \frac{1}{c^2}(\partial_\tau\hat{\Omega})^2\right) \\ & + \int_0^\beta d\tau \int d^2x \Big(\psi_{f\sigma}^*(\partial_\tau - \mu_F - iA_0)\psi_{f\sigma} + \psi_h^*(\partial_\tau - \mu_B + iA_0 + ia_0)\psi_h \\ & - \frac{1}{2m_F}\psi_{f\sigma}^*(\nabla - i\boldsymbol{A})^2\psi_{f\sigma} - \frac{1}{2m_B}\psi_h^*(\nabla + i\boldsymbol{a} + i\boldsymbol{A})^2\psi_h\Big) \\ & + \beta\sum_n \int \frac{d^2q}{(2\pi)^2} u\hat{s}(q, \omega_n)\cdot\hat{\Omega}(Q - q, -\omega_n) \\ & + \frac{1}{4i\pi}\frac{1}{2l+1}\int_0^\beta d\tau \int d^2x\, \varepsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda \end{aligned} \tag{5}$$

where $Q = (\pm\frac{\pi}{a}, \pm\frac{\pi}{a})$, $u = V_0 S\delta$, $g = 1/JS^2$, $c^2 = 8a^2J^2S^2$, $m_F = 1/t\langle\eta\rangle$, $m_B = 1/t\langle\chi\rangle$, $l = 0, 1, \ldots, \delta$ is the doping density, $\hat{\Omega}(x_i) = \eta_i\hat{S}_i/S$, $\eta_i = \pm$ is the staggered spin field, $\psi_{f\sigma}$, ψ_h the fermion fields, A_μ the gauge field which derives from the phase fluctuation of the Hubbard–Stratonovich fields η_{ij} and χ_{ij}, a_μ the Chern–Simons gauge field which derive from the nature of the hard-core boson. If we integrate out the gauge field a_μ, it is equivalent to taking the following transformation in the action:

$$\int Da_\mu D\psi_h\, e^{-S[a_\mu,\psi_h]} = \int D\phi\, e^{-S[0,\phi]} \tag{6}$$

where ϕ is the hard-core boson field.

The physical picture described by the action in (5) is clear: the spin and charge degrees of the hole are separated, but there exists an interaction between them by the gauge field. The spin part of the hole has an interaction with the staggered spin field, which mainly describes the magnetic behaviour of the normal state. The transport properties of the normal state are dominated by the charge part of the hole. The spin part of the system is mainly controlled by the corner region of the first Brillouin zone, but the charge part of the system is mainly controlled by its central region. We believe that the spin part of the system does not drastically affect its charge part and will not change its transport properties in the normal state because the staggered spin field has less influence on the hard-core boson field and gauge field. However, in the superconducting state the spin part drastically affects the charge part. The antiferromagnetic spin fluctuation tends to make the fermions pair and destroy the gauge invariance, but the gauge field is strongly pair breaking and will in general significantly suppress the pairing transition temperature. The competition of the antiferromagnetic fluctuation and gauge field fluctuation will determine the transition temperature of the superconducting state. This problem will be addressed in a separate paper. Here we only consider the physical properties of the normal state.

Generally, in the normal state, we can integrate out the fermion field $\psi_{f\sigma}$ and obtain an effective action that includes the staggered spin field, hard-core boson field and gauge field. Because of gauge invariance, the staggered spin field does not directly interact with the gauge field: they only affect each other through the fermion field. After integrating out the fermion field, we have the following effective action:

$$S_{\text{eff}} = S_{\text{eff}}^{\text{s}} + S_{\text{eff}}^{\text{c}} \tag{7}$$

$$S_{\text{eff}}^{\text{s}} = \beta \sum_n \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \left[\frac{1}{2g} \left(q^2 + \frac{1}{c^2}\omega_n^2 \right) |\hat{\Omega}|^2 - F(Q-q) \frac{|\omega_n|}{\omega_{\text{AF}}} |\hat{\Omega}|^2 \right] \tag{8}$$

$$\begin{aligned} S_{\text{eff}}^{\text{c}} = & \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^2 x \left(\psi_{\text{h}}^*(\partial_\tau - \mu_{\text{B}} + \mathrm{i}a_0 + \mathrm{i}A_0)\psi_{\text{h}} - \frac{1}{2m_{\text{B}}} \psi_{\text{h}}^* (\nabla + \mathrm{i}a + \mathrm{i}A)^2 \psi_{\text{h}} \right) \\ & + \beta \sum_n \int \frac{\mathrm{d}^2 q}{(2\pi)^2} \left(\chi_{\text{f}} q^2 + \pi \frac{|\omega_n|}{q v_{\text{F}}} \right) \left(\delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{q^2} \right) A_\alpha(q,\omega_n) A_\beta(-q,-\omega_n) \\ & + \frac{1}{4\mathrm{i}\pi} \frac{1}{2l+1} \int_0^\beta \int \mathrm{d}^2 x \varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \end{aligned} \tag{9}$$

where

$$\omega_{\text{AF}} = \frac{2v_{\text{F}}}{aN(E_{\text{F}})} \frac{1}{u^2}$$

$N(E_{\text{F}})$ is the density of states at the Fermi surface, v_{F} the Fermi wave velocity, and the factor $F(Q-q)$ can be written in the one-loop approximation as $F(Q-q) \sim \pi/a|Q-q| \sim 1$ for $q << 1$. We see that the spin and charge parts are completely separated (omitting here high-order terms). Higher-order terms may induce an interaction between the spin and charge parts, but they are irrelevant in the low-energy limit. This is true because the Fermi surface is stable and not destroyed by the gauge fluctuation due to the nature of the hard-core bosons, so one can safely integrate out the fermion field.

We now study the spin and charge parts separately. We can use the renormalization group method to study the spin part [18]. From the action (6), we see that there exist three regimes [10]. (i) In the $z = 1$ regime (where z is a dynamic exponential), the term quadratic in frequency, ω, is dominant over the linear term. (ii) In the $z = 2$ regime, the linear term is dominant. (iii) The crossover regime from $z = 1$ to $z = 2$ is determined by the last term in (6), which derives from the interaction between the copper and hole spin. We do not consider this crossover regime further.

We use the renormalization group methods developed in [4, 19] to study the behaviour of the effective action in (8). Here we adopt the symbols used in [4]: $g_0 = hc\Lambda g$, $t_0 = k_{\text{B}}Tg$, where Λ is a cutoff of the wavevector. For intermediate doping, the last term in (8) is a small quantity, which can be treated perturbatively; the frequency ω has a scaling transformation $\omega' = \omega e^l$, which corresponds to the $z = 1$ regime. In order to get the low-energy behaviour of the system we can integrate out the high-energy parts, which will induce effective coupling constants depending upon the renormalization parameter l. In the one-loop approximation, we get the following renormalization group equations of the coupling constants:

$$\frac{\mathrm{d}t}{\mathrm{d}l} = \frac{gt}{4\pi}\frac{1}{(1-a^2g^2)^{1/2}}\frac{\sinh[(g/t)(1-a^2g^2)^{1/2}]}{\cosh[(g/t)(1-a^2g^2)^{1/2})]-\cos(ag^2/t)}$$

$$\frac{\mathrm{d}a}{\mathrm{d}l} = \left(2-\frac{g}{4\pi}\frac{1}{(1-a^2g^2)^{1/2}}\frac{\sinh[(g/t)(1-a^2g^2)^{1/2}]}{\cosh[(g/t)(1-a^2g^2)^{1/2}]-\cos(ag^2/t)}\right)a \tag{10}$$

$$\frac{\mathrm{d}}{\mathrm{d}l}\left(\frac{g}{t}\right) = -\frac{g}{t}$$

where $a_0 = 1/\omega_{\mathrm{AF}}$. If we assume that the density of states at the Fermi surface obeys the relation $N(E_{\mathrm{F}}) \propto m_{\mathrm{F}}$, the quantity ω_{AF} takes the form

$$\omega_{\mathrm{AF}} \propto \frac{1}{N^2(E_{\mathrm{F}})}\frac{1}{u^2}.$$

We adopt the method used in [20] to make the renormalization group transformation for the fermion field $\psi_{\mathrm{f}\sigma}$; we have the relation $N_{\mathrm{R}}(E_{\mathrm{F}}, l) = N(E_{\mathrm{F}})\mathrm{e}^{l}$, where $N_{\mathrm{R}}(E_{\mathrm{F}}, l)$ is a renormalized density of states. This relation is correct, at least in the region $q \sim 0$.

On the other hand, for full doping, the last term in (8) is more important than the second term and we cannot treat it as a perturbative term; the frequency ω has the scaling transformation $\omega' = \omega\mathrm{e}^{2l}$, corresponding to the $z = 2$ regime, which means that the system undergoes a 'phase transition' from the $z = 1$ to the $z = 2$ regime due to the doping. In the $z = 2$ regime, the second term in (8) and high-order terms of the frequency are irrelevant under the scaling transformation; we will therefore omit these terms. Using the above methods, in the one-loop approximation we have

$$\begin{aligned}\frac{\mathrm{d}t}{\mathrm{d}l} &= \frac{gt}{4\pi}\tan^{-1}\left(\frac{g}{2t}\overline{\omega}\right)\\ \frac{\mathrm{d}}{\mathrm{d}l}\left(\frac{g}{t}\right) &= -2\frac{g}{t}\end{aligned} \tag{11}$$

where $\bar{\omega} = \omega_{\mathrm{AF}}/2g_0$ will be treated as a renormalization invariant. In fact, equations (10) and (11) are obtained in the different critical regions; they are valid only near their fixed points corresponding with the $z = 1$ and $z = 2$ regions, respectively. We can intuitively understand these problems in the following way: in the low-doping regime, the density of states $N(E_{\mathrm{F}})$ is very small, and the quantity ω_{AF} is very large; the term linear in ω_n is less important than the term quadratic in ω_n, so the system is in the $z = 1$ region. In the large-doping regime, the quantity ω_{AF} is very small; the term linear in ω_n is more important than the quadratic term, and the system is into the $z = 2$ region. However, we need a more reasonable explanation which will come from further study.

The above renormalization group equations can be easily solved if we take the quantity a to be small. Generally, for the $z = 1$ regime, according to these resolutions we can write the following expression for the spin susceptibility in the low-energy limit:

$$\chi_1(q,\omega) = \frac{\chi_0}{\xi_1^{-2} + q^2 - (1/c^2)\omega^2 - \mathrm{i}F(Q-q)\omega/\omega_{\mathrm{AF}}^{\mathrm{R}}} \tag{12}$$

where

$$\frac{1}{\omega_{\mathrm{AF}}^{\mathrm{R}}}=\frac{4\pi a^2}{k_{\mathrm{B}}T\omega_{\mathrm{AF}}(\hat{l})\xi_1^2}+\frac{1}{\omega_{\mathrm{R}}}$$

and $\xi_1 \sim 1/T$ for the QC regime, while

$$\frac{1}{\omega_{\mathrm{AF}}^{\mathrm{R}}}=\frac{16\pi a}{\omega_{\mathrm{AF}}(\hat{l})\xi_1}+\frac{1}{\bar{\omega}_{\mathrm{R}}(T)}$$

and $\xi_1 \sim$ constant for the QD regime. The terms ω_{R} and $\bar{\omega}_{\mathrm{R}}$ derive from high-order quantum fluctuations without doping [6]. In the QD regime, $\bar{\omega}_{\mathrm{R}}$ is very large: as $T \to 0$, $\bar{\omega}_{\mathrm{R}}(T) \to \infty$. In the QC regime, $\omega_{\mathrm{R}} \sim \lambda T(\xi_1/a)^2$, where λ is a constant. The term $\omega_{\mathrm{AF}}(\hat{l})$ is the renormalized quantity of ω_{AF}, where χ_0 is a constant. We see that the image term consists of two parts: one derives from the effect of undoping and another is induced by doping. For the $z = 2$ regime, we can write a general expression for the spin susceptibility in the low-energy limit:

$$\chi_2(q,\omega)=\frac{\chi_0'}{\xi_2^{-2}+q^2-\mathrm{i}F(Q-q)(\omega/\bar{\omega}_{\mathrm{AF}})} \tag{13}$$

where $\xi_2^2 \sim 1/T$ in the QC regime, and $\xi_2 \sim$ constant in the QD regime; χ_0' is a constant. These two expressions for the spin susceptibility are valid in the corner region and mainly describe the physical properties of the normal state controlled by the corner region (i.e. $Q = (\pm\pi/a, \pm\pi/a)$) of the first Brillouin zone.

Using the above spin susceptibilities we can calculate the NMR spin lattice relaxation rate T_1 and spin echo rate T_{2G} at Cu sites, which are completely determined by the real and imaginary parts of the spin susceptibility, respectively. In the QD regime, for $z = 1$ and $z = 2$, $T_{2G} =$ constant, $1/TT_1 =$ constant $+$ constant$/\bar{\omega}_{\mathrm{R}}(T)$, $T \to 0$, $\bar{\omega}_{\mathrm{R}}(T) \to \infty$. In the QC regime $T_1 =$ constant, for $z = 1$ and $z = 2$; $T_{2G} \sim T$, for $z = 1$; $T_{2G} \sim T^{1/2}$, for $z = 2$. We see that the NMR relaxation rate $1/TT_1$ increases and reaches a maximum, then decreases as T increases. In the $z = 1$ QC regime, $\xi \sim 1/T$, the renormalized density of states $N_{\mathrm{R}}(E_{\mathrm{F}}, \hat{l}) \propto N(E_{\mathrm{F}})T$. If we take the following relation for static spin susceptibility [21], $\chi(q = 0) \propto N_r(E_{\mathrm{F}}, \hat{l})$, we can explain the temperature dependence of the static spin susceptibility and the NMR spin-lattice relaxation rate at O sites, $1/TT_1 \propto \chi(q = 0)$. In the low-temperature region, the system enters the QD regime and an energy gap appears in the antiferromagnetic quantum fluctuation excitation spectrum. However, the opening energy gap drastically influences the NMR relaxation rate at Cu sites, but it has little influence on the static spin susceptibility $\chi(q = 0)$ and the NMR relaxation rate at O sites. We believe that the relation $N_{\mathrm{R}}(E_{\mathrm{F}}, \hat{l}) \sim T$ can be approximately extended to the QD regime. These results are in better agreement with current experimental data, which show that the model Hamiltonian in (1) can be better used to describe the magnetic behaviour of the normal state of cuprates. We believe that the model Hamiltonian captures the key feature of the cuprate superconducting materials, namely that the holes induced by doping have a strong magnetic correlation with the copper spin. This property is responsible for the anomalous magnetic behaviour of the cuprate superconducting materials.

In the normal state, the transport property of the system is mainly determined by the effective action (9). In the higher temperature region, the momentum relaxation rate of the fermions is $1/\tau_{tr}^{F} \propto [\max(\epsilon_k, k_B T)]^{4/3}$ and the boson transport time is $1/\tau_{tr}^{B} \propto k_B T$. We note that the electric conductivity of the system is

$$\sigma = \frac{\sigma_F \sigma_B}{\sigma_F + \sigma_B} \qquad \sigma_B = \delta \tau_{tr}^{B}/m_B \qquad \sigma_F = \delta \tau_{tr}^{F}/m_F$$

so that $\sigma_F >> \sigma_B$, $\sigma \sim \sigma_B$. The physical resistivity is dominated by the boson resistivity, which is in agreement with experiment in both the linear T dependence and the scaling of the spectral weight with hole concentration. Because of the nature of the hard-core bosons, there exists a Chern–Simons gauge field a_μ, which breaks the parity and time symmetries and induces an odd-parity gauge field propagator

$$D_{j0}(\mathrm{i}\omega, q) = \langle A_j(\mathrm{i}\omega, q) A_0(-\mathrm{i}\omega, -q)\rangle = \sigma \varepsilon_{jk} q_k F(\omega, q^2).$$

In the RPA case we can easily prove that there really exists a propagator $D_{j0}(\mathrm{i}\omega, q)$. We believe that the propagator $D_{j0}(\mathrm{i}\omega, q)$ exists over a wide energy scale, although the nature of the hard-core bosons is important in the low-energy limit. If we adopt this method and use the fermion field ψ_h instead of a boson field as in [22], we can obtain the following expression for the Hall coefficient (taking the function $F(\omega, q^2) = (1/\epsilon_0)(1/q^2)\delta_{\omega,0}$):

$$R_H = R_H^* \frac{1}{1 + \alpha T} + R_H^\infty \tag{14}$$

where $\alpha = (1/8\pi^2\epsilon_0)(n_F - n_F^2)$, $n_F = [\exp(-\mu_h/k_B T) + 1]^{-1}$ is the Fermi distribution function, and the chemical potential μ_h is proportional to the doping density δ. In the case of low doping and high temperature, the coefficient α is nearly independent of temperature. The term R_H^* is the Hall coefficient for the system without odd-parity gauge interactions; at low doping $R_H^* \propto 1/\delta$. The term R_H^∞ is the Hall coefficient in the infinite-temperature limit which can be identified [23] as

$$R_H^\infty = R_0 \left(\frac{1}{4\delta} - \frac{1}{1 - \delta} + \frac{3}{4} \right)$$

where R_0 is a constant. We believe that the charge part, which describes the transport property of the normal state, is controlled by the central region of the first Brillouin zone (which mainly reflects the character of this region of the system).

In conclusion, we have used a simple physical picture, in which holes with a single occupation constraint introduced by doping move in the antiferromagnetic background of the copper spins, to describe the normal state of cuprate superconducting materials by the traditional slave boson method. We can better explain the magnetic and transport properties of the normal state. The results obtained are qualitatively in agreement with current experimental data, which show that the model Hamiltonian captures the key feature of cuprate superconducting materials.

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